Super-Resolution from Image Sequences
A Review

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Introduction

- Seminal work by Tsai and Huang 1984
- More information in a sequence than a single frame

  Also include *a-priori* info for BW extrapolation
- SR is an ill-posed inverse problem

  (regularized solution methods needed)
- Two main classes of SR algorithm

  1. Frequency domain
  2. Spatial domain

Frequency Domain SR Methods

- Based on three principles:

  1. Shifting property of Fourier transform
  2. Alias relationship between DFT and CFT
  3. Scene is assumed bandlimited

- $\rightarrow$ system of equations relating aliased DFT coefficients of LR images to samples of the CFT of unknown scene

- Solving system $\leftrightarrow$ De-aliasing

The Details

Continuous scene: $f(x, y)$

CFT: $\mathcal{F}(u, v)$

Translations: $f_r(x, y) = f(x + \Delta x_r, y + \Delta y_r)$

CFT: $\mathcal{F}_r(u, v)$ with $r = 1, 2, \ldots, R$

Observation: $y_r[m, n] = f(mT_x + \Delta x_r, nT_y + \Delta y_r)$

$m = 0, 1, \ldots, M - 1$

$n = 0, 1, \ldots, N - 1$

DFT: $\mathcal{Y}_r[k, l]$

Aliasing:

$$\mathcal{Y}_r[k,l] = \frac{1}{T_x T_y} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \mathcal{F}_r\left(\frac{k}{MT_x} + pf_{s_x}, \frac{l}{NT_y} + qf_{s_y}\right)$$

Shifting:

$$\mathcal{F}_r(u,v) = e^{j2\pi(\Delta x_r u + \Delta y_r v)} \mathcal{F}(u,v)$$

If $f(x,y)$ is band-limited above may be combined...

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{F}$$

$\mathbf{Y}$: vector of observation image DFT's

$\mathbf{\Phi}$: matrix

$\mathbf{F}$: vector of CFT coefficients (unknown)

Solve for $\mathbf{F}$, take inverse DFT for SR image

Spatial Domain SR Methods

- Interpolation of non-uniformly spaced samples
- Iterated backprojection
- Stochastic methods
- Set theoretic methods
- Hybrid stochastic / set theoretic
- Other methods

Observation Model

Images are lexicographically ordered.

LR images: $\mathbf{y_r},\ r \in \{1, 2, \ldots, R\}$

SR image: $\mathbf{z}$

Model: $\mathbf{y_r} = \mathbf{H_r z}$

General:
$$\mathbf{Y} = \mathbf{Hz} + \mathbf{N}$$
$$\mathbf{Y} = \left[\mathbf{y_1}^T \cdots \mathbf{y_R}^T\right]^T$$
$$\mathbf{H} = \left[\mathbf{H_1}^T \cdots \mathbf{H_R}^T\right]^T$$

Noise: $\mathbf{N}$

Interpolation of non-uniformly spaced samples

- Register LR frames yielding dense composite image of non-uniformly spaced samples
- SR image reconstructed from composite
- Too simplistic
- Limited de-aliasing. Poor incorporation of *a-priori* constraints. Limited degradation models. Separate merging and restoration suboptimal.

Iterated backprojection

Simulate the LR images $\hat{\mathbf{Y}}$ as $\hat{\mathbf{Y}} = \mathbf{H}\hat{\mathbf{z}}$.

Iteratively backproject error and correct the SR estimate:

$$
\begin{aligned}
\hat{\mathbf{z}}^{(j+1)} &= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP}\left(\mathbf{Y} - \hat{\mathbf{Y}}^{(j)}\right) \\
&= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP}\left(\mathbf{Y} - \mathbf{H}\hat{\mathbf{z}}^{(j)}\right).
\end{aligned}
$$

Problems:

- Non-uniqueness of solution
- Inclusion of *a-priori* constraints difficult

Stochastic methods

- SR reconstruction as a statistical estimation problem
- Bayesian framework enables inclusion of *a-priori* info
- Stochastic observation equation $\mathbf{Y} = \mathbf{H}\mathbf{z} + \mathbf{N}$
- Maximum A-Posteriori (MAP) estimate

$$
\begin{aligned}
\hat{\mathbf{z}}_{\mathrm{MAP}} &= \arg\max_{\mathbf{z}} \left[\Pr\left\{ \mathbf{z} | \mathbf{Y} \right\} \right] \\
&= \arg\max_{\mathbf{z}} \left[\log \Pr\left\{ \mathbf{Y} | \mathbf{z} \right\} + \log \Pr\left\{ \mathbf{z} \right\} \right].
\end{aligned}
$$

Stochastic methods - MAP

$\log \Pr\{\mathbf{Y}|\mathbf{z}\}$ Log-likelihood function

$\Pr\{\mathbf{z}\}$ Prior density on $\mathbf{z}$

- $\Pr\{\mathbf{Y}|\mathbf{z}\} = f_{\mathbf{N}}(\mathbf{Y} - \mathbf{H}\mathbf{z})$ (noise PDF)
- $\Pr\{\mathbf{z}\}$ is typically a MRF
- Gaussian noise and convex priors imply convex optimization

Set theoretic methods

- Define constraint sets in space of SR image
- Solution is intersection of constraint sets
- Sets include data fidelity, positivity, bounded energy etc.
- Convex constraint sets allows use of the Projection Onto Convex Sets (POCS) algorithm
- Define constraint sets $\mathcal{C}_\alpha$ and corresponding projection operators $\mathcal{P}_\alpha$
- $\mathbf{z}^{(n+1)} = \mathcal{P}_1 \mathcal{P}_2 \mathcal{P}_3 \cdots \mathcal{P}_K \mathbf{z}^{(n)}$ (POCS)

Hybrid stochastic / set theoretic

- Best of both worlds
- Stochastic: theoretical framework, uniqueness of solution, prior densities.
- Set theoretic: convenient *a-priori* constraints
- Maximize *a-posteriori* density / likelihood function subject to satisfying convex constraint sets
- Excellent incorporation of prior info

Other methods

- Optimal and adaptive filtering
- Tikhonov-Arsenin regularization
- Few advantages not already provided by either Bayesian or POCS methods.

Frequency vs Spatial Domain SR

	Frequency Domain	**Spatial Domain**
Observation model	Frequency domain	Spatial domain
Motion models	Global translation	Almost unlimited
Degradation model	Limited, LSI	LSI or LSV
Noise model	Limited, SI	Very Flexible
SR Mechanism	De-aliasing	De-aliasing *A-priori* info

Frequency vs Spatial Domain SR

	Frequency Domain	**Spatial Domain**
Computation req.	Low	High
A-priori info	Limited	Almost unlimited
Regularization	Limited	Excellent
Extensibility	Poor	Excellent
Applicability	Limited	Wide
App. performance	Good	Good

MAP vs. POCS SR

	Bayesian (MAP)	**POCS**
Applicable theory	Vast	Limited
A-priori info	Prior PDF Easy to incorporate No hard constraints	Convex Sets Easy to incorporate Powerful yet simple
SR solution	Unique MAP estimate	Non-unique $\cap$ of constraint sets

MAP vs. POCS SR

	Bayesian (MAP)	**POCS**
Optimization	Iterative	Iterative
Convergence	Good	Possibly slow
Computation req.	High	High
Complications	Optimization under non-convex priors	Defn. of projection operators

Directions for Future Research

1. Motion Estimation

2. Degradation Models

3. Restoration Algorithms

Motion Estimation

- Holy Grail: arbitrary scenes. multiple independent motion, occlusions, transparency etc.
- Critically dependent on robust, model based, sub-pixel accuracy motion estimation
- Open research problem
- Motion estimated from the observed *undersampled* data

  – Reliability issues; Reliability measures ?

Motion Estimation

- Constrained motion estimation for consistent motion maps – Regularized motion estimation
- Sparse maps: accurate motion estimates in areas of high spatial variance (locale of best SR enhancement)
- Independent model based motion predictors and estimators
- Simultaneous multi-frame motion estimation
- Simultaneous motion estimation and reconstruction

Degradation Models

- Accurate observation model $\longrightarrow$ improved reconstruction
- Color SR – model correlations, degradations
- Lossy Compression – color subsampling, quantization, blocking effects
- Magnetic Media – recording and playback degradations
- CCD arrays – model real devices
  sensor geometry, spatio-temporal integration, noise, readout effects

Restoration Algorithms

- Hybrid MAP / POCS

  MAP – Mathematical rigor and uniqueness of solution

  POCS – Convenient *a-priori* constraints

- Simultaneous motion estimation and restoration

- Simultaneous multi-frame SR restoration